# THE PROPAGATION OF SHOCK WAVES IN VISCOUS MEDIA

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ABSTRACT: The question of the thickness of shock waves in a viscous gas was treated in papers [1, 2]. The present paper derives general equations for solving problems concerning the flow of a medium inside a shock wave layer, and the change of this layer in viscous media.

By way of an example we consider a problem of this type for a Kelvin medium.

1. It is shown in paper [3] that discontinuity waves of zero thickness cannot propagate in viscous media. We may suppose that as the result of the action of viscous forces in such media shock waves are really a layer inside which a rapid but continuous change of all functions takes place. The thickness of the shock wave layer is not the same in different places.

In deriving the fundamental equations for solving the problem of viscous flow inside the shock wave layer, we shall make the following assumptions:

a) The thickness of the shock wave layer is small and the leading and trailing fronts of the layer are parallel to each other in the first approximation.

b) If the values of some quantity z are the same on both shock wave fronts, then in the first approximation this quantity is independent of the coordinate transverse to the shock wave layer, i.e.,

$$z = C \quad \text{for} \quad [z] = 0. \tag{1.1}$$

Here C is a function independent of the transverse coordinate.

c) If the quantity z has different values on the shock wave fronts, then we shall neglect its derivatives in directions lying in the plane tangential to the fronts of the shock wave layer, compared with its derivative in the transverse direction, i.e.,

$$z_{,i} = z_{,k}\nu^{k}\nu_{i} \quad \text{for} \quad [z] \neq 0. \tag{1.2}$$

Here $\nu_i$, $\nu^k$ are the covariant and contravariant components of the normal to the shock wave layer fronts. We note that the less the thickness of the shock wave layer, the more accurate relation (1.2) becomes.

Since the shock wave layer is thin the dynamic conditions for discontinuities of density $\rho$, velocity $v_i$ and stress [4] $\sigma_{ij}$ hold,

$$[\rho(v_n - C)] = 0, \qquad [\sigma_{ij}\nu^j - \rho(v_n - G)v_i] = 0. \tag{1.3}$$

Applying (1.1) to these expressions we obtain

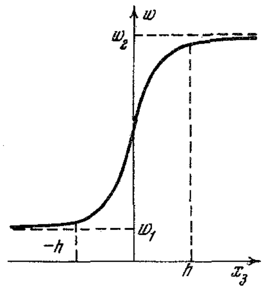
$$\rho(v_n - G) = C, \qquad \sigma_{ij}\nu^j - Cv_i = C_i. \tag{1.4}$$

Here C is an arbitrary function, $C_i$ is an arbitrary vector independent of the transverse coordinate. The first equation of (1.4) was obtained in paper [1].

If the shock wave layer only interacts weakly with the main stream of the medium, then by analogy with boundary layer theory the problem is first of all solved for the inviscid flow of a medium in which a shock wave moves. In this case dynamic, kinematic, and geometric compatibility conditions [4] are fulfilled at the shock wave. Subsequently the problem is solved for a shock wave layer where the boundary conditions on the fronts of the layer are taken to be those for a shock wave in inviscid flow.

The velocity G may be taken to be the velocity of propagation of some surface $\Sigma$ situated inside the shock wave layer and parallel to its fronts. For simplicity this surface may be taken to be the middle surface of the layer where some function assumes its mean value.

Lower indices 1 and 2 will denote the values of quantities at the trailing and leading fronts of the shock layer, respectively. Since the medium undergoes large deformations in the shock wave layer the finite nature of these deformations must be allowed for when writing down the equations governing the state of the medium in the layer.




2. In what follows we shall consider as an example the problem of the thickness of a shock wave layer and the flow inside this layer in the case of a Kelvin medium taking into account the finite magnitude of the deformations and the convective terms in determining the velocity during displacement. In this connection we shall treat the problem of the propagation of a longitudinal shock wave in an elastic medium formulated in the same way. The problem of shock wave propagation in an elastic medium where the situation is linearized was treated in paper [4].

We shall write Hooke's law in the form

$$\sigma_{ij} = \lambda e_{kk}\delta_{ij} + 2\mu e_{ij}, \qquad e_{ij} = {}^1\!/_2\,(u_{i,j} + u_{j,i} - u_{k,i}u_{k,j}), \tag{2.1}$$

where $u_i$ is the displacement vector.

In order to simplify the problem we shall choose the system of coordinates so that its origin lies on the surface $\Sigma$, and the $x_1$, $x_2$ plane coincides with the tangent plane of this surface. We introduce the notation

$$u_{1,3} = u, \qquad u_{2,3} = v, \qquad u_{3,3} = w. \tag{2.2}$$

For a longitudinal wave $[u] = [v] = 0$, $[w] \neq 0$. Determining the velocity through displacement we find in the first approximation

$$v_3 = (u_{3,t} - Gw)\,/\,(1 - w). \tag{2.3}$$

Substituting (2.1)–(2.3) into (1.3) for i = 3, we obtain an expression for determining the propagation velocity for a longitudinal wave of a strong discontinuity in an elastic medium

$$\frac{(G - u_{3,t})^2}{\lambda + 2\mu} = \frac{1 - w_2}{\rho_2}(1 - w_2)(1 - w_1)\left(1 - \frac{w_1 + w_2}{2}\right). \tag{2.4}$$

It follows from (2.4) that if $w_1$ and $w_2$ are small, then a weak longitudinal shock wave propagates with the same velocity as a longitudinal acceleration wave. If the medium is compressed on both sides of the surface ($w < 0$), then it propagates faster than an acceleration wave. If the medium is stretched on both sides of the wave, its propagation velocity is less than the velocity of an acceleration wave. For very strong extension when $w_2$ or $w_1 \to 1$, the propagation velocity of the shock wave decreases to zero. For strong compression when $w_2$ or $w_1 \to -\infty$, the shock wave velocity increases without limit.

3. We shall write the relation between the stress and deformation tensors and the velocity for a Kelvin medium in the form [5]

$$\sigma_{ij} = (\lambda e_{kk} + \xi\varepsilon_{kk})\,\delta_{ij} + 2\mu e_{ij} + 2\eta\varepsilon_{ij}. \tag{3.1}$$

We have from (2.3)

$$v_{3,3} = \frac{(1 - w)\,w_{,t} - (G - u_{3,t})\,w_{,3}}{(1 - w)^2}. \tag{3.2}$$

Simultaneously solving the system of equations (1.4), (2.3), (3.1), and (3.2), and taking (1.2) into account, we obtain the following nonlinear equation for the function w:

$$(1-w)\{aw^3-3aw^2+[C_3+a(2+u^2+v^2)+$$
$$+C(Gw-u_{3,t})]w-[C_3+a(u^2+v^2)]\}=$$
$$=(\xi+2\eta)[(G-u_{3,t})w_{,3}-(1-w)w_{,t}] \qquad (a={}^1/_2\lambda+\mu). \tag{3.3}$$

We shall assume that the viscosity does not affect the flow at the leading and trailing fronts of the shock wave. In this case $w_1$ and $w_2$ are the roots of the polynomial on the left side of (3.3). Equation (3.3) may be written in the form

$$a(1-w)(w-w_1)(w-w_2)(w-w_0)=$$
$$=(\xi+2\eta)[(G-u_{3,t})w_{,3}-(1-w)w_{,t}]. \tag{3.4}$$

If $w_1w_2 \neq 0$, then the root is

$$w_0=\frac{C_3+a(u^2+v^2)}{aw_1w_2}. \tag{3.5}$$

In the case of loading or unloading waves $w_1w_2 = 0$ the root $w_0$ is

$$w_0=\frac{C_3+a(2+u^2+v^2)+C(Gw-u_{3,t})}{aw^*},$$
$$w^*=\begin{cases}w_2 & (w_1=0)\\ w_1 & (w_2=0)\end{cases}. \tag{3.6}$$

In order to find an expression for the root $w_0$ in terms of $w_1$ and $w_2$, we shall find the values of $C_3$ and C in terms of quantities at the shock wave. Setting $w = w_2$ in the left side of (3.3) and equating it to zero we obtain

$$C_3=2aw_2-aw_2^2-a(u^2+v^2)+\frac{C(Gw_2-u_{3,t})}{1-w_2}w_2. \tag{3.7}$$

From the first equation of (1.4) and (2.3) we have

$$C=-\rho_2(Gw_2-u_{3,t})/(1-w_2). \tag{3.8}$$

Substituting (3.7), (3.8), and (2.4) into (3.5) and (3.6), respectively, we obtain

$$w_0=3-(w_1+w_2), \qquad w_0=3-w^*. \tag{3.9}$$

Thus in both cases (3.5) and (3.6) the difference $w_0 - w = 3 - (w + w_1 + w_2)$ is positive. The left side of (3.4) inside the shock wave layer is positive.

Equation (3.4) may be integrated in the quasi-stationary state when $w_{,t} = 0$. We shall write the boundary condition in the form

$$w={}^1/_2(w_1+w_2) \quad \text{for} \quad x_3=0. \tag{3.10}$$

When (3.4) is integrated with the condition (3.10), we obtain for the case under consideration

$$x_3=\frac{(G-u_{3,t})(\xi+2\eta)}{a}\times$$
$$\times\left[a_1\ln\frac{2(1-w)}{2-w_1-w_2}+a_2\ln\frac{3(2-w_1-w_2)}{2(w_0-w)}+\right.$$
$$\left.+a_3\ln\frac{w_2-w_1}{2(w_2-w)}+a_4\ln\frac{2(w-w_1)}{w_2-w_1}\right],$$
$$a_1=\frac{1}{(w_0-1)(1-w_1)(1-w_2)},$$
$$a_2=\frac{1}{(w_0-1)(w_0-w_1)(w_0-w_2)},$$
$$a_3=\frac{1}{(1-w_2)(w_0-w_2)(w_2-w_1)},$$
$$a_4=\frac{1}{(1-w_1)(w_0-w_1)(w_2-w_1)}. \tag{3.11}$$

Equation (3.11) determines the variation of the quantity w between the values $w_1$ and $w_2$. For $x_3 \to \pm\infty$ the quantity w rapidly approaches the asymptotic values of $w_2$ and $w_1$ (Fig. 1). The main change in w occurs over a distance of the order

$$h=\frac{8(G-u_{3,t})(\xi+2\eta)}{3a(2-w_1-w_2)^2(w_2-w_1)}, \tag{3.12}$$

which may be regarded as the thickness of the shock wave layer.

It follows from (3.12) that this thickness decreases according as the shock wave intensity is larger, the greater is the compression on the medium on one side of the shock wave, or the smaller is the coefficient of viscosity. Thus very strong shock waves in a Kelvin medium can be treated as waves with no thickness. Weak shock waves in a quasi-static process are very thick.

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